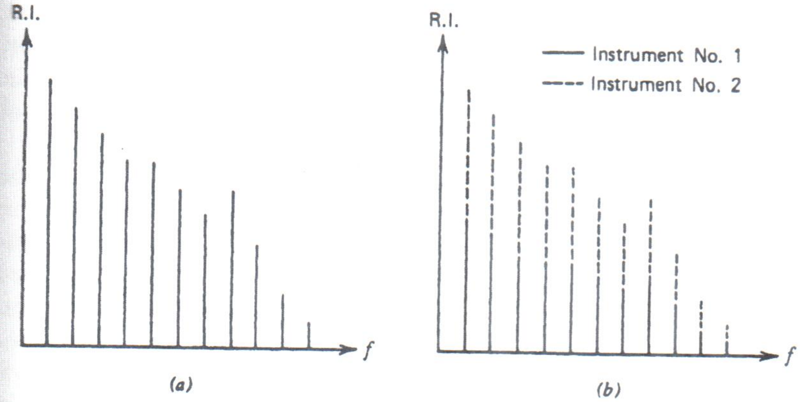

Figure 13.1
(*a*) A complex tone from one instrument. (*b*) A complex tone resulting from adding a complex tone from one instrument (– – –) to the complex tone of a second instrument (———).

The perception of musical sound is a complicated process. Suppose, for example, that a complex tone reaching the ear can be represented by the audio spectrum shown in Figure 13.1*a*. Such a tone is perceived as having a characteristic timbre determined, we say, by the harmonic structure of the tone. Suppose, however, that the stimulus shown in Figure 13.1*a* is in actual fact the consequence of *two* different instruments, each with their own harmonic structure, playing a tone of the same pitch. The audio spectra of the two instruments (Figure 13.1*b*) combine to give a stimuli equivalent to the spectrum of the single tone illustrated in Figure 13.1*a*. Nevertheless, our auditory system perceives the spectrum shown in *b* as *two* separate tones while spectrum *a* is perceived as *one* tone! How our auditory system accomplishes this feat is a mystery.

There are many questions concerning musical perception for which answers are tenuous at best. The problem of consonance and dissonance is one of the most critical areas in the subject of musical perception. To this topic we now direct our attention.

CONSONANCE AND DISSONANCE

If two tones played together are judged by the average listener as pleasant, the tonal combination is referred to as a consonance. On the other hand, if the combination is judged harsh, jarring, or unpleasant, it is a dissonance. Clearly, these terms have reference to subjective judgments; however, as we shall see, the basis for these judgments is a shifting basis and therefore makes an objective analysis difficult.

Dissonance is important in music. A musical score made up entirely of consonant intervals would probably be judged bland or even insipid by the mature listener. Dissonance brings a restlessness to a movement that is resolved in consonance. In the same way that suspense and conflict bring tension to drama, dissonance brings tension to music. Without this tension, the relaxation brought by consonance would be empty of meaning.

Music has become increasingly dissonant. Why is this? It is because the subjective basis of dissonance varies with time. A tonal combination introduced in one age can strike the musical audiences of that generation as harsh and discordant. But, with time, audiences get accustomed to it and that dissonant interval becomes more and more consonant. Consequently, a later generation of composers has to create new dissonances to produce the same degree of tension as their predecessors. Compare, for example, Mozart's *Eine Kleine Nachtmusik* composed in 1787 with Arnold Schoenberg's *Vorgefuhle* composed in 1909. Such a comparison reveals that the history of music has been characterized by the emancipation of the dissonant interval.

Musical tradition and the listener experience circumscribed by that tradition is one dimension of the consonance–dissonance problem. Musical training, another kind of experience, is another dimension. Still another dimension emerges from the fact that a dissonant interval is more acutely dissonant when played by two similar instruments than when it is played by two different instruments.

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These considerations should serve to illustrate the subjective nature of consonance and dissonance. Now let us see what objective considerations can be brought to bear on the subject.

The Beat Theory of Helmholtz

A complex tone is characterized by its harmonic structure. When two complex tones are played together, the harmonics of each tone are present in the stimulus arriving at the ear of the listener. For some tonal combinations the harmonic frequencies match each other exactly. For others they do not.

1. The Unison

Let us consider a perfect unison. Figure 13.2 shows that the harmonic frequencies of each tone match each other exactly. The unison is pleasant to the ear and is considered a consonant.

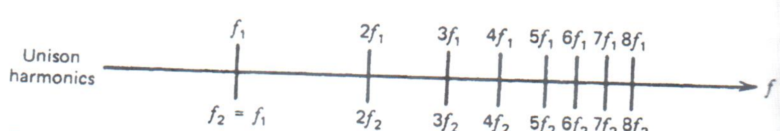

Figure 13.2
Harmonics of the unison.

2. The Whole-Note Interval

The situation changes with the whole-note interval. The ratio of frequencies between whole notes (on the tempered scale) is 1.123. If two complex tones a whole note apart are played together, the sound is unpleasant and is classed as a dissonance. The harmonic structure of each tone is shown in Figure 13.3. There is an obvious mismatch for all frequencies. Furthermore, the frequencies are close enough together so that discernable beats as well as harshness can result. The beat theory of Helmholtz maintains that it is the beats that our auditory system responds to negatively and as a result the whole-note interval is unpleasant.

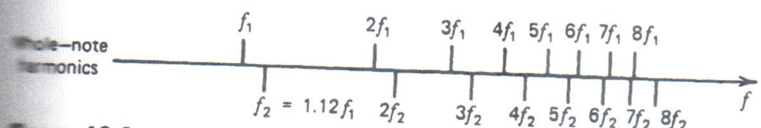

Figure 13.3
Harmonics of the whole-note interval.

3. The Perfect Fifth

If complex tones of frequencies f and $\frac{3}{2}f$ are played together, the interval is a musical fifth. The harmonics of these two tones, illustrated in Figure 13.4, show some matching frequencies and some potentially "colliding" frequencies.

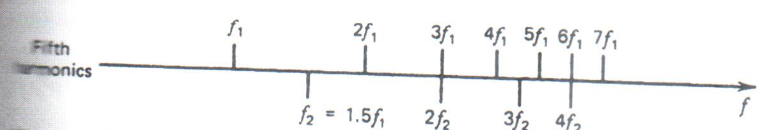

Figure 13.4
Harmonics of the musical fifth.

The third harmonic of f_2 is between the fourth and fifth harmonics of f_1. With the fifth, however, there are enough frequencies that match or are separated by a margin wide enough to be troublefree. The interval is therefore "beat free," and is a consonance.

Other musical intervals can be analyzed in similar fashion. An indication of how dissonant a musical interval is can be judged by how far along the harmonic sequence one has to go before a matchup of harmonic frequencies occurs. Table 13.1 shows various musical intervals

TABLE 13.1
Musical Intervals

Interval	Frequencies	First Pair of Matching Harmonics
Unison	f_1 and $f_2 = f_1$	f_2 with f_1
Octave	f_1 and $f_2 = 2f_1$	f_2 with $2f_1$
Fifth	f_1 and $f_2 = \frac{3}{2}f_1$	$2f_2$ with $3f_1$
Fourth	f_1 and $f_2 = \frac{4}{3}f_1$	$3f_2$ with $4f_1$
Third (major)	f_1 and $f_2 = \frac{5}{4}f_1$	$4f_2$ with $5f_1$
Sixth (major)	f_1 and $f_2 = \frac{5}{3}f_1$	$3f_2$ with $5f_1$
Third (minor)	f_1 and $f_2 = \frac{6}{5}f_1$	$5f_2$ with $6f_1$
Sixth (minor)	f_1 and $f_2 = \frac{8}{5}f_1$	$5f_2$ with $8f_1$
Whole tone	f_1 and $f_2 = 1.123f_1$	No match up
Semitone	f_1 and $f_2 = 1.0595f_1$	No match up

and identifies the first pair of matching harmonic frequencies. As one proceeds down the table the number of colliding harmonics increases and the degree of dissonance increases. In the beat theory, the former is considered the cause of the latter.

Table 13.1 can be summarized as follows. A musical interval can be expressed as the ratio of two frequencies; that is,

$$\frac{f_2}{f_1} = \frac{n}{m}$$

where for most intervals n and m are integers. In general, the smaller the numbers, the greater the degree of consonance. Or, in terms of the beat theory, the small numbers imply an absence of disturbing beats among
FFT 2 the higher harmonics.

There are numerous difficulties with the beat theory. An example or two will serve to illustrate these difficulties. It is reasonable to assume that a change in timbre, that is, a change in the relative intensities of the harmonics, would bring an alteration in the intensity of the resulting beats. Yet, a change in timbre does not alter the judgment of consonance or dissonance. In other words, a consonance produced by violins is also a consonance when it is produced by French horns. Furthermore, consider a major third and a major second. The harmonics of the major third C_3 to E_3 and the major second C_4 to D_4 are shown in Figure 13.5. Included in Figure 13.5 are the frequency differences between corre-
FFT 3 sponding harmonics. The critical differences occur between the respec-
FFT 4 tive fundamentals and the second harmonics. As can be seen, they are
FFT 5 essentially the same. Yet, the major third is considered a consonance while the major second is a dissonance.

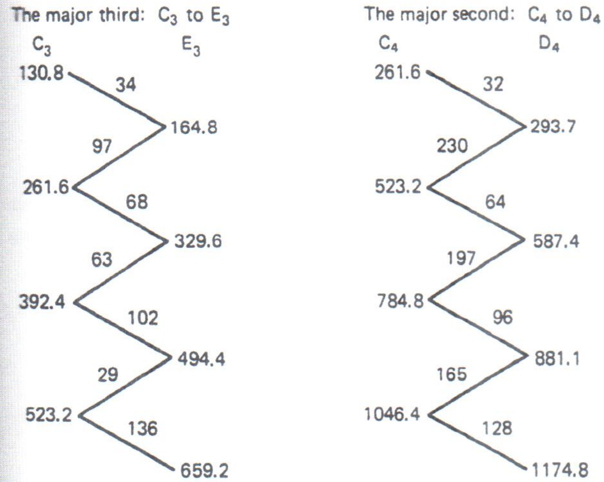

Figure 13.5
Harmonics of the major third C_3–E_3 and the major second C_4–D_4. The former is a consonance and the latter a dissonance.

The Critical Band Theory of Plomp

Let us begin by considering pure tones. Specifically, we shall consider a fifth, two major thirds, and a major second. The fifth C_4 to G_4 is a consonance. The major thirds C_4 to E_4 and C_3 to E_3 are interesting because the former is definitely a consonance while the latter is marginally a consonance or perhaps even a dissonace. The major second C_4 to D_4 is a dissonance.

The fifth C_4 to G_4 involves frequencies of 262 and 392 Hz (assuming the tempered scale). The center frequency, that is, the average frequency, is 327 Hz. The critical bandwidth at 327 Hz is 100 Hz. This means that any two tones within the range 277 to 377 Hz will have a harsh, rough quality. The frequencies 262 and 392 Hz do not fall within this range.

With a series of experiments on consonance and dissonance judgment, Plomp has concluded that two pure-tone frequencies falling outside the critical band are judged consonant. Figure 13.6 illustrates the case of the fifth as being consonant. These experiments further reveal that the *most* dissonant interval is the one for which the two pure-tone frequencies are separated by 25 percent of the critical band. In fact, frequencies differing in the range from 5 to 50 percent of their corresponding critical band are typically judged dissonant. Therefore, we are led to conclude that two pure tones whose frequency difference is less than 50 percent of the appropriate critical bandwidth comprise a dissonance.

For the major third C_4(262 Hz) to E_4(330 Hz), the center frequency is 296 Hz for which Δf_{CB} = 95 Hz. The major third C_3(131 Hz) to E_3(165 Hz) has a center frequency of 148 Hz with Δf_{CB} = 90 Hz. Finally, the ma-

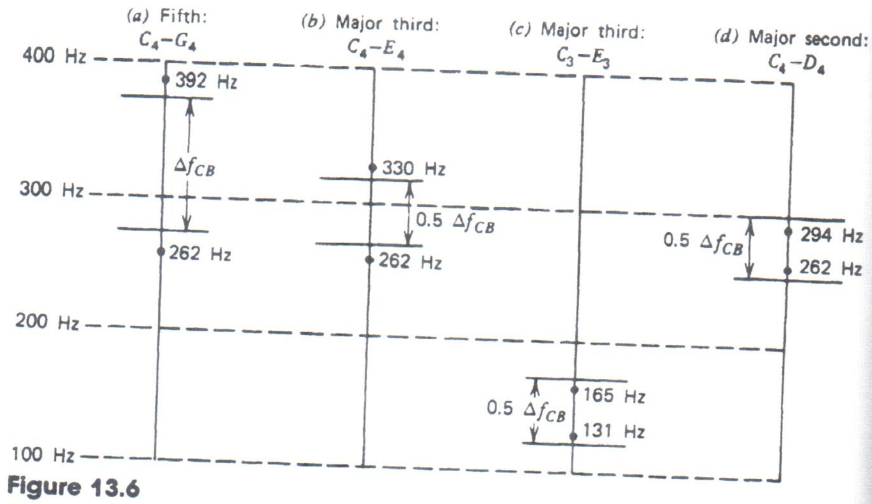

Figure 13.6
Two pure-tone frequencies form four different musical intervals. In terms of the Plomp criterion the first two are consonants while the latter two are dissonants.

jor second C_4(262 Hz) to D_4(294 Hz) has a center frequency of 278 Hz with $\Delta f_{CB} = 95$ Hz. Figure 13.6 illustrates each of these intervals. The major third C_4 to E_4 is a consonance since the frequency difference of the two pure tones is greater than $0.5\Delta f_{CB}$. One octave lower in frequency, however, the major third C_3 to E_3 is certainly less consonant in terms of the Plomp criterion and may even be considered a dissonant interval. The major second C_4 to D_4 is certainly a dissonance.

The experimental results obtained for pure tones can now be extended to complex tones. Each complex tone in a musical interval has a family of harmonics. If harmonics up to f_6 are considered for each complex tone, consonances will have a predominance of neighboring harmonics whose frequency difference is greater than $0.5\Delta f_{CB}$. A dissonance, on the other hand, will have neighboring harmonics whose frequency difference is less than $0.5\Delta f_{CB}$. (See Figure 13.7.)

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FFT 7

MUSICAL SCALES

There was music long before there were any scales. People sang and did so with a sequence of musical pitches that brought satisfaction to their ears. Likewise, primitive musical instruments were made that could generate those musical sounds that were desired by the instrument makers.

Over the centuries came the scales, many different musical scales in different cultural settings as well as a succession of scales within any one musical tradition. Composers wrote music, and scales evolved from the music as it was performed.

Intonation describes the way that musical intervals are played when there is a freedom to play those intervals so that they sound the best to

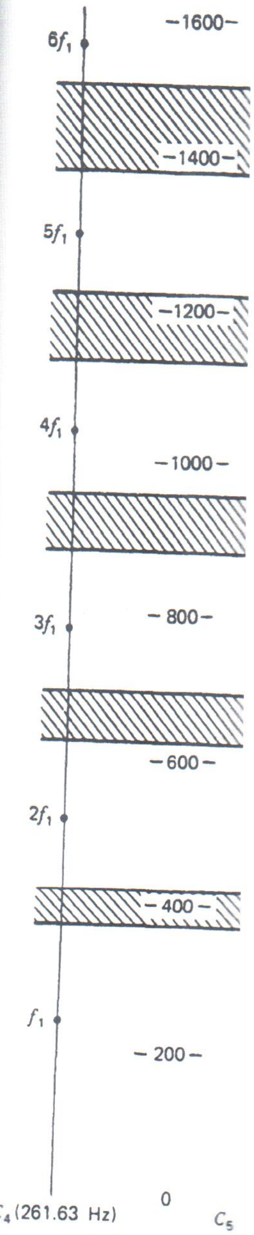

Figure 13.7
An octave (left) and a
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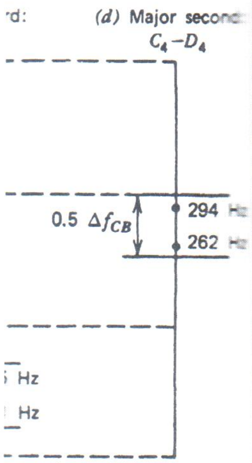
third A_3 to C_4 is more



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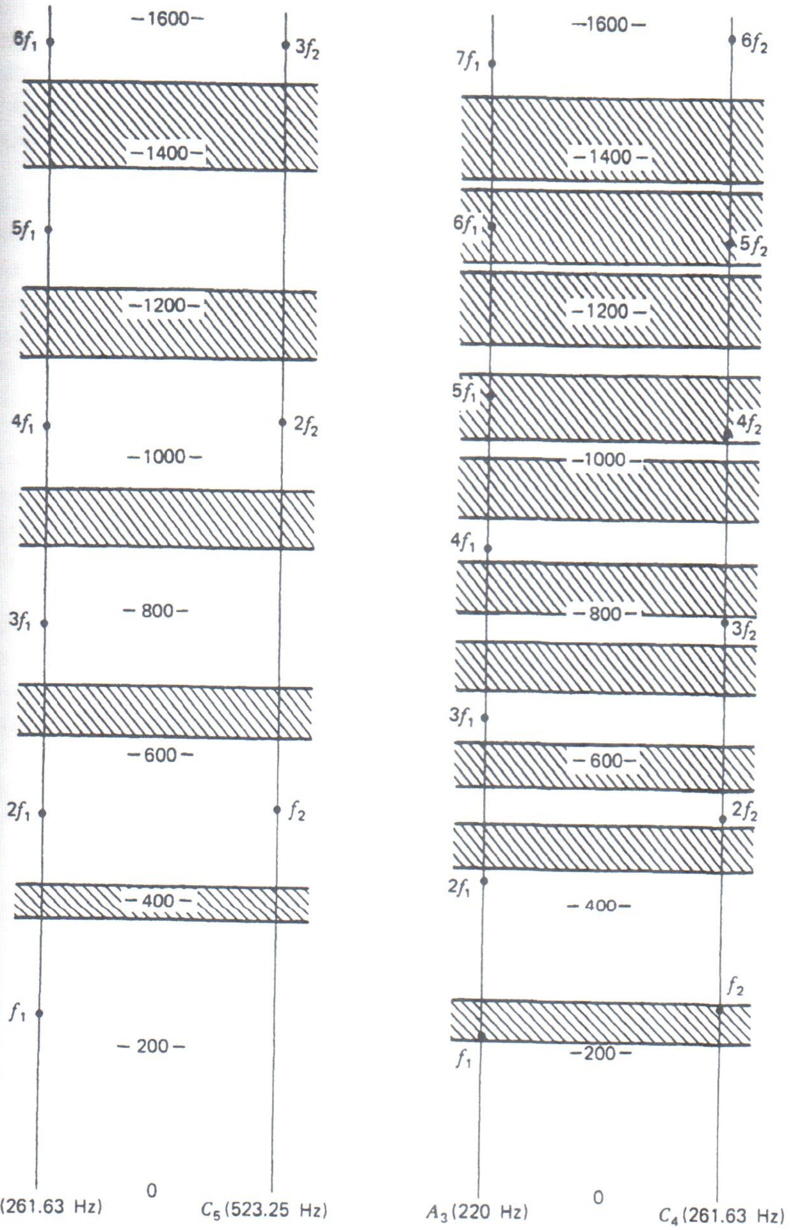

Figure 13.7

An octave (left) and a minor third (right) are illustrated. The shaded areas are centered on the average frequency between adjacent harmonics and they cover a frequency band equal to $0.5\Delta f_{CB}$. Note that for the minor third f_1 and f_2, $5f_1$ and $4f_2$, $6f_1$ and $5f_2$ fall within a shaded area. For the octave, all harmonic frequencies are outside the shaded areas. By the Plomp criterion, the minor third A_3 to C_4 is more dissonant than the octave C_4 to C_5.