Assignment 10 Solutions

Page 253-254:

4. Analyze (1) an octave and (2) a perfect fourth, and, in terms of the beat theory of Helmholtz, show which harmonics might be expected to produce beats.

(1) The octave is essentially the same as a perfect unison because the higher frequency in the octave interval has a fundamental which falls exactly on the second harmonic of the lower frequency and all the higher harmonics of the upper frequency (both odd and even) fall exactly on the even harmonics of the lower frequency. So unless the octave is mistuned, there are no beats to be heard.

(2) For a fourth the lower frequency is the 3rd harmonic of a some fundamental and the upper frequency is the 4 harmonic. The ratio of the upper to lower frequency is $4/3$. The first perfect match is $3f_2$ with $4f_1$. Except for the very low registers, $f_1$and f_2 are separated by more than the critical band so beats will not play a role there, but we must also consider the higher harmonics of each frequency between f_2 and the first unison at $3f_2 = 4f_1$. Here is a specific example. Take $f_1 = C_4 = 262Hz$. The harmonics including C_4 are: 262, 524, 786, 1048, ... Here $f_2 = 349.33$ with higher harmonics 698.67, 1048. So we have a possible conflict between 699 Hz and either 524 Hz or 786 Hz. $786 - 699 = 77$ Hz, and $699 - 524 = 175$ Hz. 175 Hz is not a problem, but 77 Hz is well within the critical band and can be expected to cause some dissonance or roughness in the sound due to beats according to the Helmholtz theory of beats.

5. Consider the following complex tones that together form a musical interval. Interval 1: 130.8 and 164.8 Hz. Interval 2: 261.6 and 329.6 Hz. In terms of the beat theory, which interval would be the more consonant? Both of the these intervals are major thirds. Interval 1 is the $C_3 - E_3$ major third, and interval 2 is the $C_4 - E_4$ major third. Both intervals will have the same potential conflicts because they are both major thirds, but the lower interval's potential conflicts will be closer in frequency and therefore, more likely to have a disonant sound due to beats. Hence, the upper interval, interval 2 is the more consonant. More specifically if we follow the general outline of the analysis in question 4, we would consider harmonics between164.8 Hz and $5f_1 = 654$ or $4f_2 = 659.2$ Hz. This is already a problem, because they are slightly out of tune in equal temperment and would produce a fused tone with audible beats. The other frequencies are 164.8, 261.6, 329.6, 392.4, 494.4, 523.2, For interval 2 we start with 329.6 and go up to $5f_1 = 1308$ or $4f_2 = 1318.4$, also out of tune. The harmonics are 329.6, 523.2, 659.2, 784.8, 988.8, 1046.4. The differences for the lower interval are: 96.8, 68, 62.8, 102, 28.8, and for the upper interal are: 193.6, 136, 125.6, 204, 57.6. We see that the upper third's differences are exactly twice the differences produced by the lower major third and are more consonant.

6. The following paris of tones are played with pure tones. Use the Plomp cirterion to determine whether they are a consonance or a dissonance.

(a) $D_6 - B_5$. $D_6 = 1174.7$ Hz and $B_5 = 987.77$ Hz. They differ in frequency by 186.9 Hz. At the average frequency of 1081.2 Hz, $\Delta f_{CB} = 180$ Hz which is less than the frequency difference between

D_6 and B_5. Thus, this is a clear consonance by the Plomp criterion.

(b) $D_7 - C_7$. D_7 =2349.3 Hz and C_7 =2093.0 Hz. They differ in frequency by 256.3 Hz. At the average frequency of 2221.1 Hz, $\Delta f_{CB} = 320$ Hz. Since 256.3 Hz is less than Δf_{CB}, the sound will not be completely smooth, but since the frequency difference is greater tha 50% of Δf_{CB}, by the Plomp criterion, it is a consonance.

7. Perfect fourths have frequency ratios of 4/3. The frequencies of three different perfect fourths are given below. In terms of the Plomp ccriterion, which of these fourths would be the most consonant? The least consonant? We analyzed a perfect fourth in question 4 above so we know where the potential conflicts can arise. In this case the 1st fourth is at low frequency and the two frequencies are at 65.4 and 87.3 Hz respectively which are separated by only 22 Hz which is well within 50% of Δf_{CB}. As we go up in frequency each successive fourth is more consonant with respect to the Plomp criterion and that goes for the conflicts between the higher harmonics as well. Thus, we conclude that the 3rd fourth is the most consonant and the 1st fourth is the least consonant.

8. In question 3 (last weeks assignment), you established that between the notes A_0 and A_7 there are 7 octaves and 12 musical fifths. If we assume that the frequency of A_0 is f,

(a) What is the frequency of A_7 regarded as *exactly* 7 octaves above A_0? Using this perscription, $A_7 = A_0 \times 2^7 = f \times 2^7 = 128f$.

(b) What is the frequency of A_7 regarded as *exactly* 12 fifths above A_0? Using this perscription $A_7 = A_0 \times \left(\frac{3}{2}\right)^{12} = f \times \left(\frac{3}{2}\right)^{12} = 129.746...f$

THEY ARE NOT EQUAL!

12. In question 8 you found that the circle of fifths does not close. Obviously, the same notecannot have two different frequencies. Interms of the temperament of the piano, what compromise is made. The piano is tuned to equal temperament which means that the ratio of every semitone interval is exactly the same. Call this ratio a where a is slightly larger than one. Since there are 12 semitone intervals in an octave we have $f \times (a)^{12} = 2f$. Canceling f on both sides leads to $a = (2)^{\frac{1}{12}} = 1.059463... \sim 1,0595$. Since the fifth is 7 semitones above the tonic of a scale $f_2 = a^7 \times f_1 = 2^{\frac{7}{12}} f_1 = 1.4983$ compared with the theorectical for a perfect fifth of 1.5. So all the fifths on a piano are slightly flat.

13. Assume a musical score calls for the playing of a major third. It is to be played by a violin and a clarinet. Based on your understanding of consonance and dissonance, would you have the violin play the lower frequency or the higher frequency? Keeping in mind the fact that the sound spectrum for the clarinet is missing the even harmonics because of its construction, there is less chance of the higher harmonics of the clarinet forming dissonant matches with the higher harmonics of the violin if the clarinet plays the lower note in the major third. Also the 5th harmonic of the clarinet will match the 4th harmonic of the violin if the clarinet plays the lower note, but not otherwise. This is a good thing and should provide a pleasing renforcement for the major 3rd being played. In the opposite case with the clarinet on top there are dissonances between the 3rd harmonic of the clarinet and 4th harmonic of the violin and between the 5th harmonic of the clarinet and 6th and 7th harmonics of the violin. These conclusions are shown in the table below where we have chosen the major 3rd to start on concert A, first with the Clarinet on the bottom and then with the violin.

clarinet		Violin		comment	Violin		Clarinet		comment
1	440				1	440			
		1.25	550				1.25	550	
		2.5	1100		2	880			
3	1320				3	1320			
		3.75	1650	$> \Delta f_{CB}$ OK			3.75	1650	
5	2200	5	2200	matched	4	1760			$< 50\% \; \Delta f_{CB}$
		6.25	2750		5	2200			no match
7	3080				6	2640			
		7.5	3300				6.25	2750	$< 50\% \; \Delta f_{CB}$
					7	3080			$< 50\% \; \Delta f_{CB}$