Assignment 4 Solutions

Page 77:

2. Two pure tones with frequencies of 220 and 228 Hz are played together. A microphone is placed at a point where waves from both sources are detected.

(a) What is the time interval between *successive* compressions? Here the effective frequency we hear is a fused frequency which is the average of 220 and 228 Hz or 224 Hz. Successive compressions occur once per period which is the reciprocal of 224 Hz or 0.00446 seconds $= 4.46 \times 10^{-3}$ seconds.

(b) The amplitudes of successive compressions are different. What is the time interval between compressions having maximum amplitudes? Since we have two sine waves close in frequency, they go in and out of phase and then back in phase at the beat frequency which is the difference between the two frequencies. The time we are looking for is the reciprocal of this beat frequency or 1 divided by 8 Hz $=$ 0.125 seconds.

3. Answer Question 2 for the case where the two frequencies are 212 Hz and 220 Hz.

(a) Now the frequencies are 212 Hz and 220 Hz so the fused freqency is 216 Hz which gives a time between successive compressions of $1/216$ Hz $= 4.63 \times 10^{-3}$seconds.

(b) However, the beat frequency is the same as exercise 2 so that answer to finding the time between successive compressions of maximum amplitude is the same, namely, 0.125 seconds.

5. Consider a violin string of length 0.316 m. Waves travel on tis string with a velocity of 277 m/s.

(a) What is the largest period a wave can have if it is to be accommodated by the string as a standing wave? Since the string is fixed at both ends, the resonance wavelengths are given by the relationship $\lambda = {}^{2L}\!/\!_{n}$ where L is the length of the string and n is an integer. The maximum λ occurs when $n = 1$corresponding to the lowest frequency which gives the largest period. $\lambda = 2 \times L = 2 \times 0.316m = 0.632m$. The period

$$\tau = 1/f = \frac{1}{{}^{v}\!/\!_{\lambda}} = \frac{1}{{}^{277m/s}\!/\!_{0.632m}} = \frac{0.632m}{277m/s} = 2.28 \times 10^{-3}s.$$

(b) From the formula for the possible wavelengths at resonance we see that possible frequencies are integer multiples of the lowest frequency so the successive periods are given by

$$\tau_n = \frac{1}{nf} = \frac{\tau}{n} = (1.14, 0.76, 0.57, 0.46) \times 10^{-3}s$$

where $n = 2, 3, 4, 5$, for example.

6. The thing to take away from this question is that the distance between the nodes or between the antinodes of a standing wave pattern is equal to $1/2$ of a wavelenth. Let distance between nodes or between antinodes be ΔL. Then the wavelength $\lambda = 2\Delta L$.

7. The internodal distance of a standing wave on a string is 0.45 m. The frequency of the wave is 800 Hz. With what velocity does a wave propagate along the string? Using the result from exercise 6 we see that wavelength $\lambda = 2 \times 0.45m = 0.90m$. If the frequency $f = 800$ Hz, then $v = \lambda f = 0.9m \times 800s^{-1} = 720m/s$.

8. A string is 0.80 m long. It is fastened securely at both ends: at $x = 0$ m and $x = 0.80$ m.

(a) Are the points $x = 0$ m and $x = 0.80$ m vibrational nodes or vibrational antinodes?

The definition of a node is a point that is not moving in the standing wave pattern. Since the ends of the string are fixed, they must be nodes.

(b) A wave with $\lambda = 0.20$ m forms a standing wave on the string. Is the point $x = 0.40$ m a vibrational node or antinode? Sketch the standing wave pattern. The internodal distance is $\frac{\lambda}{2} = 0.10$ m, so there are nodes every 0.1 m starting at $x = 0$. Clearly, $x = 0.40$ m is a vibrational node. So your sketch should look like Fig. 6.6 on page 77 with 9 nodes starting at $x = 0$ and ending at $x = 0.80$ m.

Pages 86-87:

1. Two tones having frequencies of 512 and 516 Hz are layed together.

(a) What is f_{amp}? f_{amp} is the frequency of the amplitude envelope which is equal to twice the beat frequency Δf_B. So $f_{amp} = \frac{(516-512)}{2} = 2$ Hz.

(b) What is Δf_B? The frequency difference between the two tones is 4 Hz. This is the number of times/second that the two tones go from in phase to in phase per second or the number of beats/second that we hear: $\Delta f_B = 4$ Hz.

(c) Are beats perceived? At four beats/second, they will be very clear. They will start to be perceived as just a rough sound when the beat frequency exceed 15 Hz.

2. An oboe is playing a 440 A. A second oboe plays the same note and six beats are heard each second. If we assume the first instrument's tone has a frequency exactly equal to 440 Hz:

(a) What is the frequency of the second oboe? All we know is that the frequency difference between the oboes is 6 Hz. So the second oboe could have a frequency of either 434 Hz or 446 Hz.

(b) Is the second oboe sharp or flat with respect to the first oboe? We can't tell from the information given which of the two frequencies is actually being played.

The rest of the questions have to do with reading the graph in Fig. 7.1 on page 82. The qualification that the tones are played softly means that there are no aural harmonics due to nonlinearity of the auditory system because the sounds are loud enough for the ear to distory them.

3. A pure tone with frequency 1000 Hz is played. A second pure tone, higher in pitch, is played with the first tone. If two distinct tones are perceived, what is the smallest possible frequency of the second tone? If two distinct tones are perceived it means we are in region III of Fig. 7.1 above the limit of frequency discrimination Δf_D. At 1000 Hz, $\Delta f_D \cong 67$ Hz, therefore, the second pure tone would have to be at least 1067 Hz for two pure tones to be heard. Below the limit of frequency discrimination, we would hear a fused tone which for this case is about 1033 Hz, suggesting that we should bump up the threshold frequency to say between 1070 and 1075 Hz.

4. Two pure tones having frequencies of 1000 and 1040 Hz are played together softly. What is perceived? Since at 1000 Hz, $\Delta f_D \simeq 67$ Hz, the two tones of 1000 Hz and 1040 Hz are too close together in frequency to be perceived as separate tones. Rather they are perceived as a fused tone of 1020 Hz which is heard as a rough sound because of the beats.

5. Two pure tones having frequencies of 1500 and 1550 Hz are played together softly, What is

perceived? At 1500 Hz, $\Delta f_D \simeq 125$ Hz, therefore, the two tones of 1500 and 1550 Hz are perceived as a fused tone of 1525 Hz which is heard as rough because of the beats.

6. Two pure tones having frequencies of 500 and 650 Hz are played together softly, What is perceived? Because the frequency difference between 500 Hz and 650 Hz is 150 Hz, we are clearly above Δf_D which is much less than 100 Hz at these frequencies. Therefore, two pure tones are heard. Furthermore, because $\Delta f_{CB} \simeq 110$ Hz and $\simeq 120$ Hz at frequencies of 500 and 650 Hz, respectively; a frequency difference $f = 150\,Hz > \Delta f_{CB}$ means that we hear the pure tones as smooth where the roughness from the beats has disappeared.

7. At a frequency of 750 Hz:

(a) What is Δf_D? From Fig. 7.1, at 750 Hz, $\Delta f_D \simeq 44$ Hz.

(b) What is Δf_{CB}? Again from Fig. 7.1, at 750 Hz,$\Delta f_{CB} \simeq 133$ Hz.

(Please note that contrary to the author's claim otherwise, the upper and lower curves are not drawn as to be perfectly symmetric with respect to the unison line. I marked my own book using the lower half, but careful investigation shows that the upper curves can easily be interpreted to by approximately 10 Hz higher around 1000 Hz.)