Assignment 5 Solutions

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4. One of the strings of a piano is 96 cm in length. If the string is vibrating in its sixth vibrational mode,

(a) How many antinodes exist along the string? For a string fixed at both ends the number of antinodes equals the harmonic number. Since the string is vibrating in its sixth vibrational mode, the number of antinodes is six.

(b) What is the wavelength of the standing wave on the string? for a string fixed at both ends the wavelength is given by $\lambda = {}^{2L}\!/_{n}$where L is the length of the string and n is the number of the vibrational mode. So $\lambda = {}^{2L}\!/_{n} = \frac{2\times 0.96m}{6} = 0.32m$.

6. A string has a linear density of 0.035 kg/m. If it is stretched to a tension of 667 N, what is the velocity of the wave on the string? For a string under tension the velocity is given by $v = \sqrt{\frac{T}{d}} = \sqrt{\frac{667N}{0.035kg/m}} = 138.0m/s$, where T is the tension in Newtons and d is the linear density in kg/m.

7. Consider the string in the previous question. If the string is 1.0 m long, what is the frequency of the fourth resonant mode? For the fourth resonant mode, $\lambda = \frac{2L}{n} = \frac{2\times 1.0m}{4} = 0.5m$. The frequency $f_4 = \frac{v}{\lambda} = \frac{138.0m/s}{0.5m} = 276Hz$.

8. A harp string is 52.0 cm long and it is tuned to a frequency of 660 Hz. Find the wavelength

(a) of the fifth harmonic. $\lambda = {}^{2L}\!/_{n} = \frac{2\times 0.52m}{5} = 0.208m$.

(b) of the sound waves in the air at 22 degrees Celsius that results from the fifth harmonic. Here the wavelength of sound in air is not simply related to the wavelength on the string. However, the string wave and the sound wave in air both have the same frequency. Since the fundamental of the string is 660 Hz, the fifth harmonic has a frequency of $5\times 660Hz = 3300Hz$. The velocity of sound in air is given by $v = 20.1\sqrt{T_A} = 20.1\sqrt{22+273} = 345.2m/s$. Now we use $\lambda = \frac{v}{f} = \frac{345.2m/s}{3300Hz} = 0.105m$.

9. A stretched string vibrates with a frequency of 440 Hz (A_4on the piano). If the tension in the string is doubled, what is the new pitch? Here we combine the relationships $f = \frac{v}{\lambda}$ and $v = \sqrt{\frac{T}{d}}$ to write $f = \frac{1}{\lambda}\sqrt{\frac{T}{d}}$. If we double the tension, the new frequency $f_2 = \frac{1}{\lambda}\sqrt{\frac{2T}{d}} = \sqrt{2}\times\frac{1}{\lambda}\sqrt{\frac{T}{d}} = \sqrt{2}\times f = 1.414\times 440Hz = 622.2Hz$.

17. Let us assume an instrument has strings of length 76 cm. Part of the string is suspended over a fingerboard. When the string is pressed to the finger board:

(a) What happens? The length of the string is shortened. The new end is where the string is pressed to the finger board.

(b) When the string is fingered a distance of 8 cm from the end, the string vibrates with a frequency of 300 Hz. where would the finger have to be placed to obtain a vibration frequency of 311 Hz? We know that the string with a length $L_1 = 76cm - 8cm = 68cm$ vibrates with a frequency of 300 Hz. The relationship to use here is $f = \frac{v}{\lambda}$. We write $f_1 = \frac{v}{\lambda_1} = 300Hz$ and $f_2 = \frac{v}{\lambda_2} = 311Hz$, from which we form the ratio $\frac{\lambda_2}{\lambda_1} = \frac{f_1}{f_2} = \frac{300Hz}{311Hz} = 0.9646$. Now $\lambda_1 = \frac{2L_1}{1} = 136cm$. Therefore, $\lambda_2 = \lambda_1 \frac{f_1}{f_2} = 136cm \times 0.9646 = 131.2cm = 2L_2$. Thus, L_2 =65.6cm and the finger must be placed $76 - 65.6cm = 10.4cm$ from the end.