Assignment 6 Solutions

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8. Two pure tones having frequencies of 440 and 587 Hz are played together loudly. What are the frequencies of the combination tones most likely to be perceived? Due to aural distortion of the ear due to loud sounds, the combination tones most likely to be heard are $f_2 - f_1$, $2f_1 - f_2$, $3f_1 - 2f_2$. Using the frequencies given, these are: $f_2 - f_1 = 587Hz - 440Hz = 147Hz$, $2f_1 - f_2 = 2 \times 440Hz - 587Hz = 293Hz$, $3f_1 - 2f_2 = 3 \times 440Hz - 2 \times 587Hz = 146Hz$.

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11. An open cylindrical tube is 0.05 m long. If you ignore the end correction factor, what would be the frequency of the third harmonic? First we need to find the speed of of sound in dry air at $10°C$. This is given by the formula $v = 20.1\sqrt{T_A} = 20.1 \times \sqrt{273 + 10} = 338.1m/s$. If we don't use an end correction factor the wavelength of the first harmonic is given by $2L = 2 \times 0.05m = 0.10m$. The frequency of the 3rd harmonic is given by $f_3 = 3 \times f_1 = 3 \times \frac{v}{\lambda_1} = 3 \times \frac{338.1m/s}{0.10m} = 10144Hz$.

15. A semiclosed cylindrical tube has a length of 0.5 m and a radius of 6 cm. What is the frequency of this tube's first harmonic? This tube only has a correction at the single open end which is $0.613 \times 6cm = 3.678cm$. Therefore, the effective length of the tube is $L_{eff} = 0.5m + 0.01878m = 0.53678m$. The fundamental wavelength $\lambda_1 = 4L = 4 \times 0.53678m = 2.1471m$. Using the velocity of sound in dry air at room temperature of 345m/s, we have $f_1 = \frac{v}{\lambda_1} = \frac{345m/s}{2.1471m} = 160.7Hz$. This differs slightly from the result in the book.

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1. A string is plucked at the midpoint. What is the intensity of the ninth harmonic relative to the fundamental? As explained in the text, the intensity goes as $\frac{1}{n^2}$, therefore, the ninth harmonic is reduced in intensity by 1/81 from the fundamental.

2. A stretched string is 60 cm long. It is plucked with a plectrum at a point 20 cm from one end.

(a) What harmonics would you expect to be in the resulting vibration? Because it is plucked one third of the way from the end, the 3rd, 6th, 9th, etc. harmonics are missing. We would expect to have 1, 2, 4, 5, 7, 8, 10 and so on.

(b) Indicate what the sound spectrum might look like. We would expect the intensity for the hamonics present to fall off as $\frac{1}{n^2}$.

3. A flexible string of length 99 cm and mass 0.001 kg is stretched to a tension of 110 N.

(a) What is the linear density of the string? The linear density d is given by $d = \frac{m}{L} = \frac{0.001kg}{.99m} = 1.01 \times 10^{-3}kg/m$.

(b) If the string is plucked with a sharp object at a point 24.25 cm from one end, what are the four smallest harmonic frequencies in the resulting vibration? Because the pluck point is very close to one quarter of the distance from one end, the 4th harmonic will be essentially absent. That leaves 1, 2, 3, and 5 as the 4 smallest harmonics. The velocity along the string is given

by $v = \sqrt{\frac{T}{d}} = \sqrt{\frac{110N}{1.01\times10^{-3}kg/m}} = 330m/s$. The fundamental wavelength is $2L = 1.98m$, and the fundamental frequency is $f = \frac{v}{\lambda} = \frac{330m/s}{1.98m} = 167Hz$. Thus, the 1st, 2nd, 3rd, and 5th harmonics are 167, 333, 500, and 833 Hz respectively.

4. The figure in the text shows the initial shape of a plucked string. The plucking point is spread out between 12 and 15 cm from one end of a string 60 cm in length. Which harmonics would be absent from the complex vibration? Since the plucking point is spread out over a distance from one end ranging between one fifth and one quarter of the length of the string, we know that the 4th and 5th harmonics and all integer multiples of these two are absent from the vibration frequency spectrum. That leaves 1, 2, 3, 6, 7, 9, 11 and so on. We will be missing 4, 5, 8, 10, 12, 15, 16, 20 and so on.

7. Consider a stretched string 30 cm long. If the string is up-bowed, what time does a point 6 cm from one end spend rising? Express your answer as a percentage of the period of oscillation. Since 6 cm is one fifth of the length of the string, this point will spend one fifth of its time slipping and four fifths of its time rising. Thus, the time rising is 80% of each period or 0.8 T.