Assignment 8 Solutions

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3. The open D-string on a violin is tuned to a frequency of 294 Hz.

(a) What would be the desirable freqauency of the MAR: The discussion of the violin suggests that violin should be constructed so that the "main air resonance" MAR is just below the frequency of the D string (by no more than a whole tone). A semitone would be about 17 Hz lower, for example. Since the book shows an answer of approximately 285 Hz we will use that for the next part.

(b) Let's use this frequncy to estimate the volume of the enclosed air in a good violin. To carry out this estimation, assume that the thickness of the top plate at the f-holes is 0.25 cm and that the area of each f-hole is 6.5 cm^2. What is the calculatd volume? Does your result seem reasonable?

Here we use the formula for a Helmholtz resonator which is $f = \frac{v}{2\pi}\sqrt{\frac{A}{lV}}$ which after some algebra can be written as $V = \frac{A}{l}\left(\frac{v}{2\pi f}\right)^2 = \frac{13.0\times10^{-4}m^2}{0.25\times10^{-2}m}\left(\frac{345m/s}{2\pi 285Hz}\right)^2 = 0.0193m^3$. This is of the order 20,000 cm^3 which would correspond to a rectangular box 100 cm by 40 cm by 5 cm and seems barely plausible.

5. The frequency of the air resonance is given by the formula $f = \frac{v}{2\pi}\sqrt{\frac{A}{lV}}$. If all the dimensions of the cello were exactly twice those of a violin, show that the air resonance of a cello would be one-half that of a a violin. In physics we call this making a dimensional argument. If the size grows by a factor of 2, it means that the length grows by 2, the area area by $2^2 = 4$, and the volume by $2^3 = 8$. Putting these factors into the square root part of the formula produces the expected factor of 1/2 since $\sqrt{\frac{2^2}{2\times2^3}} = \frac{1}{2}$.

12. Assume the Bflat clarinet has an overall length of 67 cm.

(a) Based on this actual length, what should be the lowest frequency of the clarinet? Using our model of a cylindrical tube closed at one end, the fundamental wavelength of the instrument is $4\times 67cm = 2.68m$. This gives a frequency $f = \frac{v}{\lambda} = \frac{345m/s}{2.68m} = 128.7Hz$ or rounding off – 129Hz.

(b) The lowest note played by the Bflat clarinet is D$_3$. What is the effective length of the clarinet? The frequency of D$_3$ is 146.8 Hz. We use our formula $\lambda = \frac{v}{f} = \frac{345m/s}{146.8Hz} = 2.35m$. This corresponds to an effective length $L_{eff} = \frac{\lambda}{4} = \frac{2.35m}{4} = 0.587m \backsimeq 59cm$.

(c) How do you account for the answers in (a) and (b)? First, when I measured my own clarinet including the mouth piece, I got 66 cm rather than 67 cm, but that is pretty close. The difference is due to effects of the bell and the mouthpiece whose effective lengths are less than their actual lengths. There is also some shortening due to slight increase in diameter to correct for the increasing size of the tone holes.

16. Let us assume that the length of the trombone is 270 cm when the slide is in the first position. How far would the slide have to be moved out to lower the pitch by four semitones? For

each semitone we must increase the length by 6% or a factor of 1+0.06. We need to do this 4 times, once for each semitone. Therefore, $\Delta l = 270cm \times (1 + 0.06)^4 - 270cm = 70.86cm$. To get this increase in length the slide must be move out by 1/2 this distance which is 35.4 cm.

18. The third valve of a trumpet adds a length of tube to lower the pitch by three semitones. (Valve 3 alone is more-or-less equivalent to valves 1 and 2 together.) Consider the situation when all three valves of the trumpet are depressed. What discrepancy would you predict between the actual and ideal lengths? With all three values depressed, we have a trumpet whose length has been increased by $2 \times \left[X_0 \times (1 + 0.06)^2 - X_0 + X_0 \times (1 + 0.06) - X_0\right]$. The term $X_0 \times (1 + 0.06)^2 - X_0$ is the increase due to valve 1 alone and the term $X_0 \times (1 + 0.06) - X_0$ is the increase due to valve 2 alone. The sum of the two terms is the increase due to valve 3 alone. Therefore, if we have all three valves depressed we have the increase given. However, we need a trumpet whose length has been increase by $X_0 \left(1 + 0.06\right)^6 - X_0$. If we assume that $X_0 = 140$ cm (see book), then the two lengths are 51.4 cm and 58.6 cm. Our trumpet is too short by 7.2 cm.

22. A piano string is 140 cm long with a linear density of 0.042 kg/m. The string is stretched to a tension of 2000N. The hammer is designed to strike the string a distance equal to 1/7 of its length from one end. What is the maximum time that the hammer can be in contact with the string if we assume that the hammer must break contact with the string before the wave reflected from the far end of the string returns to the point of hammer impact. The first thing we need to know is the velocity of the wave on the string as given by the formula $v = \sqrt{\frac{T}{d}} = \sqrt{\frac{2000N}{0.042kg/m}} = 218.2m/s$. 1/7 of the length is 20 cm, so the distance to the far end is $140cm - 20cm = 120cm$. The distance to go there and back is 240 cm. This takes a time $t = \frac{distance}{v} = \frac{2.40m}{218.2m/s} = 0.011s$ or $11ms$. Thus, this is the maximum time the hammer may be in contact with the string.