Assignment 9 Solutions

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2. A musical interval can be expressed as a ratio: $\frac{f_2}{f_1} = \frac{n}{m}$. What is n and m for

(a) a musical fifth?

(b) a major third.

The harmonic series consists of the integer multiples of a fundamental frequency starting with 1, then 2, 3, and so on. The frequency intervals between the successive integers have names. The interval $f_1$to f_2 is an octave, the interval f_2 to f_3 is a fifth, the interval f_3 to f_4 is a fourth, the interval f_4 to f_5 is a major third, and the interval f_5 to f_6 is a minor third. So for a fifth $\frac{f_3}{f_2} = \frac{3}{2}$, and for a major third $\frac{f_5}{f_4} = \frac{5}{4}$.

3. Sart at the lowest note on a piano, A_0, and play upward from that note for seven successive octaves.

(a) What note do you end up on? If we go up an octave from an A_0, we land on an A so we increase the subscript by one and call the note A_1. If we go seven successive octaves from A_0, we will be at A_7.

(b) How many musical fifths are between these starting and ending points? It is probably easier to do this while sitting at a piano keyboard, but no matter. Here are the chromatic keys in an octave: C, $C,^{\#}$ D, $D,^{\#}$ E, F, $F,^{\#}$ G, $G,^{\#}$ A, $A,^{\#}$ B, C. Keeping in mind that there are 3 whole tones and one semitone making up the interval of a fifth, the so called circle of fifths starting on A_0 is: A_0, E_1, B_1, $F_2^{\#}$, $C_3^{\#}$, $G_3^{\#}$, $D_4^{\#}$, $A_4^{\#}$, F_5, C_6, G_6, D_7, A_7. From this sequence of fifths, we see that after 12 successive fifths we land back on an A, in this case A_7. So 12 successive fifths must be the same as 7 octaves. As we shall see in the solutions for the next assignment, if we use the harmonic ratios given above to define the octave and the fifth, there is no way for 7 octaves to equal 12 fifths. This will mean that building any musical scale is going to be a compromise!